CARINGBAH HIGH

2021 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- Reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2-6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7-26)

- Attempt Questions 11– 40
- Allow about 2 hours and 45 minutes for this section

Marker's Use Only						
Section I	**Section II**					**Total**
Q1-10	**Q11-18**	**Q19-24**	**Q25-30**	**Q31-35**	**Q36-40**	
/10	/18	/18	/18	/17	/19	/100

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Section I

10 marks
Attempt Question 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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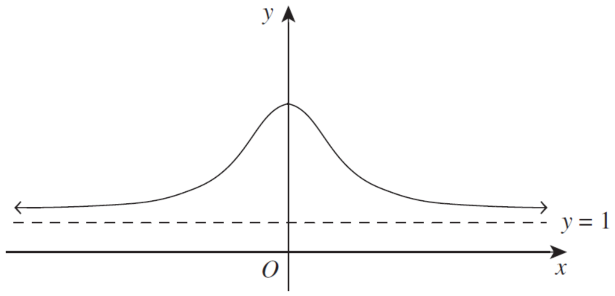
1 A geometric sequence has a first term of 20 and a common ratio of -1.5, find the 5th term.

(A) 12.5 (B) $-151\frac{7}{8}$ (C) $101\frac{1}{4}$ (D) $-101\frac{1}{4}$

2 Find the domain for $y = \dfrac{2x}{\sqrt{x+2}}$

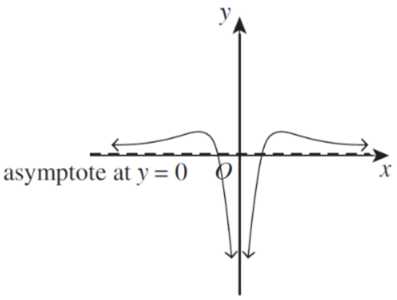
(A) $[-2,\infty)$ (B) $(-\infty,-2]$ (C) $(-2,\infty)$ (D) $(-\infty,-2)$

3 A sketch of the function $y = f(x)$ is shown below with a horizontal asymptote at $y = 1$.



Which of the following could be the sketch of $y = f'(x)$?

(A)



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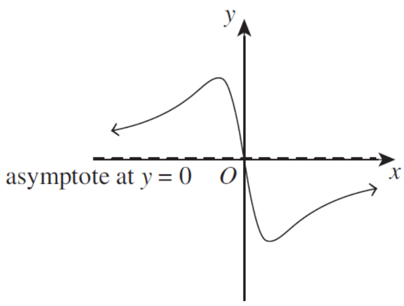

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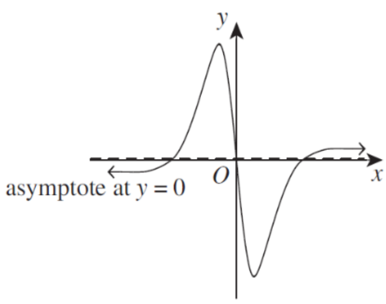
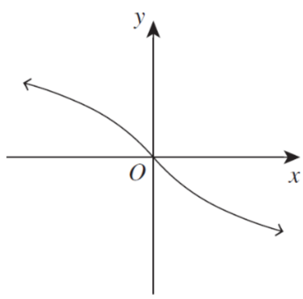


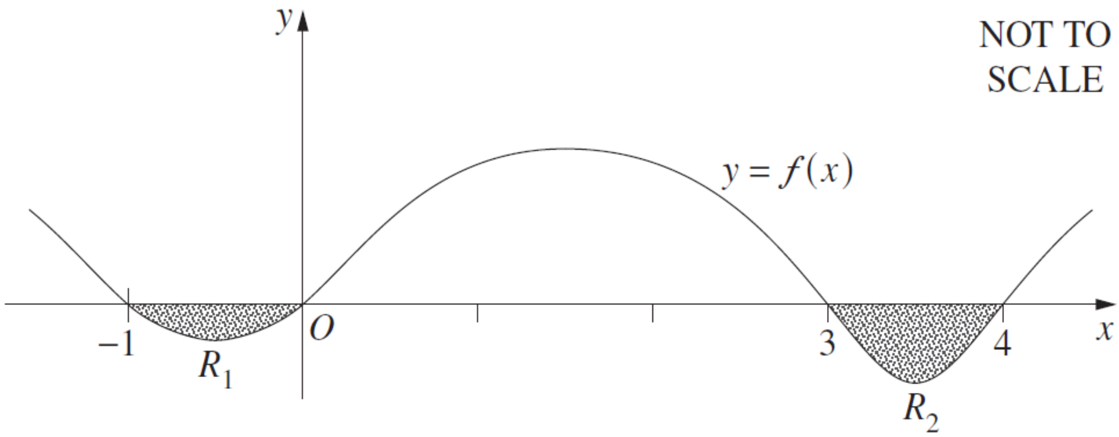
(D)



4 What is the derivative of $\cos(\ln x)$, where $x > 0$?

(A) $-\dfrac{\sin(\ln x)}{x}$ (B) $\dfrac{\sin(\ln x)}{x}$ (C) $-\sin\left(\dfrac{\ln x}{x}\right)$ (D) $-\sin(\ln x)$

5 The diagram shows the graph of $y = f(x)$ with intercepts at $x = -1$, 0, 3 and 4.



The area of the region shaded R_1 is 2 square units.
The area of the region shaded R_2 is 3 square units.

It is given that $\int_0^4 f(x)\,dx = 10$

What is the value of $\int_{-1}^3 f(x)\,dx$?

(A) 5 (B) 9 (C) 11 (D) 13

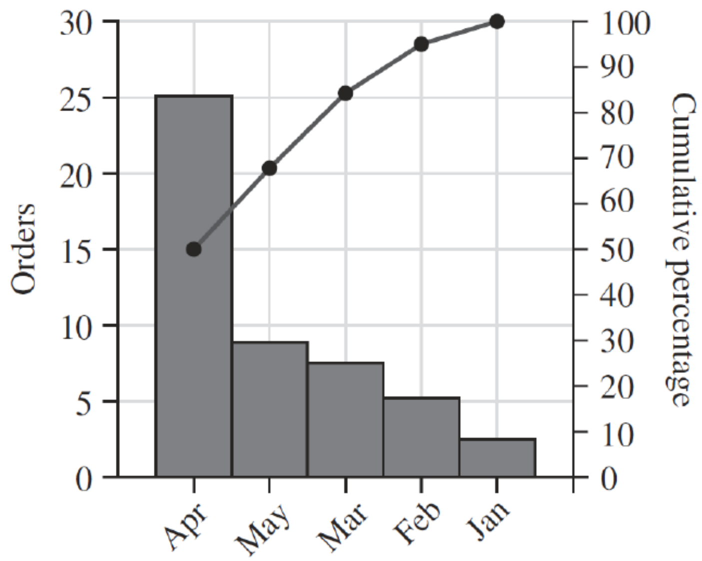
6 The number of lollies in a packet are normally distributed. 95% of lolly packets have between 23.5 and 26.7 lollies. Find the size of the standard deviation.

(A) 25.1 (B) 1.6 (C) 3.2 (D) 0.8

7 A particle is moving is moving with velocity $v = t^2 - 10t + 21,\ t \geq 0$. The particle is stationary when:

(A) $t = 3$ (B) $t = 5$ (C) $t = 3$ or 7 (D) $t = 5$ or 7

8 The following Pareto chart shows the orders that Isabella made for her company during a five month period.



Approximately what percentage of the orders occurred in May?

(A) 69% (B) 45% (C) 30% (D) 19%

9 Which statement is true for an ungrouped data set with no outliers?

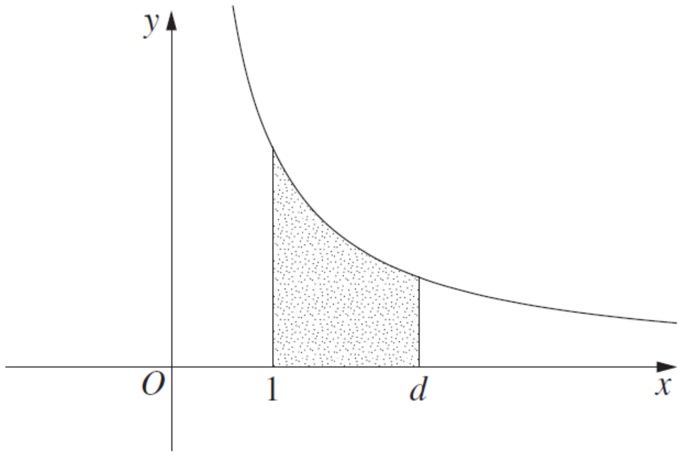
(A) The largest possible range is 2 times the interquartile range

(B) The largest possible range is 3 times the interquartile range

(C) The largest possible range is 4 times the interquartile range

(D) The largest possible range is 5 times the interquartile range

10 The diagram shows the area under the curve $y = \frac{2}{x}$ from $x = 1$ to $x = d$.



What value of d makes the shaded area equal to 2?

(A) e (B) $e + 1$ (C) $2e$ (D) e^2

End of Section I

CARINGBAH HIGH

2021 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11–40
Allow about 2 hours and 45 minutes for this section

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- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
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Marks

Question 11

Jasper records the number of Year 12 students who studied for at least one hour each day for each of the past ten days, the results were as follows:

15, 18, 20, 20, 22, 26, 28, 32, 34, 47

a) Find the mean correct to one decimal place. **1**

b) Find the interquartile range. **1**

c) Is 47 an outlier for this set of data? Justify your answer with calculations. **1**

Question 12

Evaluate $\sum_{r=1}^{6} r^r$ **1**

Question 13

Annabelle takes 2 cans, without replacement, from a refrigerator that contains 8 cans of Pepsi, 4 cans of Coke and 3 cans of Sprite. Find the probability that Annabelle gets:

a) 2 of the same drink **1**

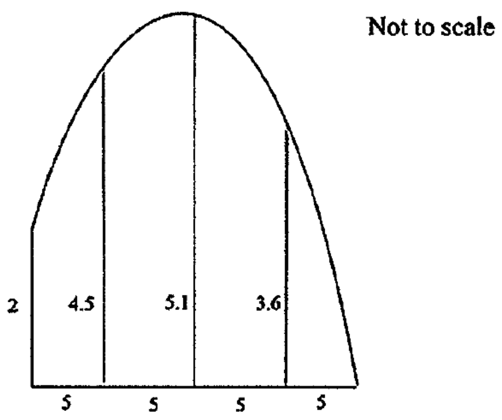
b) At least one Pepsi **1**

Question 14

Solve $|7x-12| \geq 16$ **2**

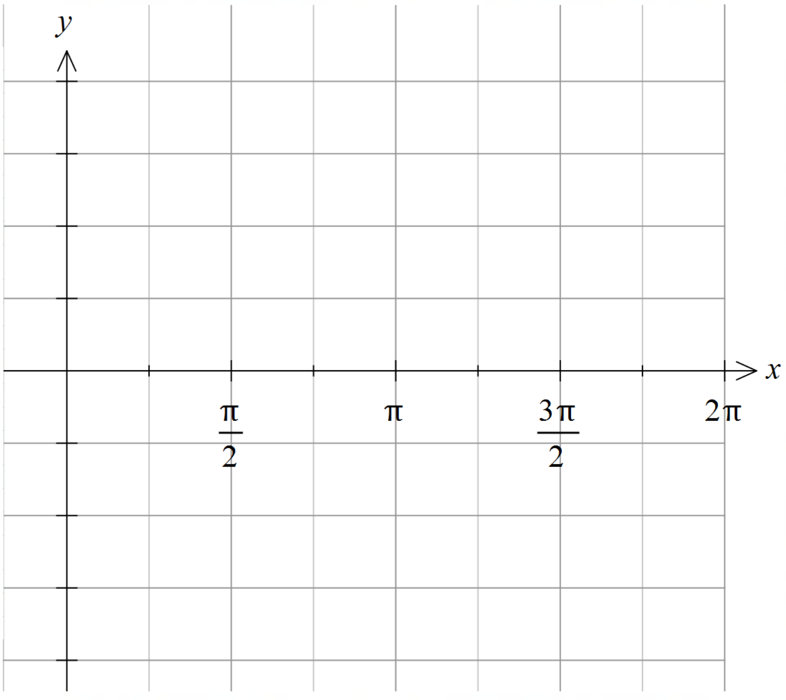
Question 15

Ronan is planting trees in the native garden which has dimensions as shown in the diagram. He can plant one tree per 3 square metres. Using the trapezoidal rule with 4 intervals, find the approximate area of the garden and therefore determine the number of trees he can plant. **2**



Question 16

Sketch the graph of $y = 3\sin 2x + 1$ for $0 \le x \le 2\pi$ **2**



Question 17

Jure owns a building supply company. They sell packets of screws which claim to have 75 screws per packet. To avoid customer complaints, the mean amount of screws in a packet is 79 and the variance is 4.

a) What is the probability that a packet of screws selected randomly will have less than 75 screws in it? **1**

b) The business sets a target of less than 16% of all blocks will have more than 80. The business can change the mean weight of the boxes to meet this target. What is mean amount of screws required to meet the target if the standard deviation is 2? **1**

Question 18

a) $\int \cos x^\circ dx$ **2**

b) $\int_1^2 \frac{x}{2x^2+1} dx$ **2**

Question 19

Find $\dfrac{dy}{dx}$ and leave each answer as a single simplified fraction:

a) $y=(3x-4)\sqrt{5-2x}$ **2**

b) $y=\dfrac{\sin x}{1+\cos x}$ **2**

Question 20

The table below shows Luke's marks as well as the cohorts' mean and standard deviation across Science and English exams **2**

Subject	**Luke's Mark**	**Mean**	**Standard Deviation**
Science	86	73	8
English	83	74	5.5

In comparison to the cohort, in which of the subjects did Luke perform better? Justify your answer with relevant mathematical calculations.

Question 21

Find $\int_{2}^{3} e^{5-2x}\,dx$, expressing your answer as a single simplified fraction **2**

Question 22

Jasmine's salary in her job increases by the same amount every year. In the 4th year at her job she earned \$81 000. In the 9th year of her job she earned \$98 500.

a) Find how much Jasmine earned in the first year of her job and by how much her pay increases each year. **2**

b) How much will Jasmine have earned in total after working at her job for 15 years? **1**

c) Bree started a job at the same time, and with the same starting salary as Jasmine. Bree's salary, however, increased by 2% per annum. With the use of logarithms, show in which year will her income first be greater than \$100 000? **2**

Question 23

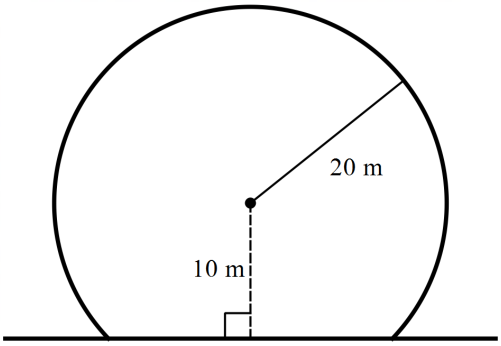
Prove that $\dfrac{\cos\theta}{1-\sin\theta}-\tan\theta=\sec\theta$ **3**

Question 24

A school has two History classes that sit an exam. Andy's class has 16 students and has a mean mark of 65%, whilst Juliana's class has 24 students. The combined mean for the 2 classes is 68.6%. Find the mean for the Juliana's class. **2**

Question 25

Peter ties his dog to a post with a 20 m long rope. On one side, the post is 10 m from a fence. Calculate the area correct to 4 significant figures, that the dog can play. **2**



Not to scale

Question 26

Use the table of values of $\phi(z)$ below to find $P(-0.9 \le Z \le 1.4)$ **2**

	first decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

You are halfway through the exam (50 marks out of 100)

Question 27

A continuous random variable, T, represents the time taken in days to show symptoms after contracting a virus. It has the following probability density function.

$$f(t) = \begin{cases} \dfrac{k}{2t-1} & \text{for } 1 \le t \le 14 \\ 0 & \text{for } 0 \le t < 1 \text{ or } t > 14 \end{cases}$$

a) Show that $k = \dfrac{2}{\ln 27}$ **2**

b) After how many days will a person have a 75% chance of showing symptoms after they have contracted the virus? **2**

Question 28

Kyle recorded the number of push-ups and the number of sit-ups each of his classmates could do in a minute, as seen in the table below.

Push-Ups	Sit-Ups
8	18
10	17
17	22
22	30
29	25
36	47
40	50
48	48
51	57
60	81

a) The value of the correlation coefficient (r) is 0.95, explain what this means in the context of his data set. **1**

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b) Use your calculator to find the equation of the least-squares regression line in the form of $y = Bx + A$. (Round each number to 1 decimal place.) **1**

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c) If Kyle is able to perform 32 sit-ups, use your equation in part b) to calculate the expected number of push-ups he could perform (to the nearest whole number). **1**

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Question 29

Show that $\frac{d}{dx}\left[\ln\sqrt{\frac{2+x}{2-x}}\right]=\frac{2}{4-x^2}$ **3**

Question 30

A drug is used to control a medical condition. It is known that the quantity Q milligrams of drug remaining in the body after t hours satisfies an equation of the form $Q=Q_0e^{kt}$ where Q_0 and k are constants. The initial dose is 6 milligrams and after 15 hours the amount remaining in the body is half the initial dose.

a) Find the values of Q_0 and k. **2**

b) After how long will one-eighth of the initial dose remain? **2**

Question 31

Kloe has taken up golf, each year she plays 30 rounds of golf. In 2011 she totalled 3300 shots, in 2021 she totalled 2760 shots. The number of shots she took each year decreased by the same amount. To make it as a professional golfer she needs to get under 2115 shots in her 30 rounds. If this trend continues, in what year can she become a professional golfer? **3**

Question 32

a) Show that the derivative of $y = \log_e(\tan x)$ is $\dfrac{dy}{dx} = \tan x + \cot x$ **2**

b) Hence find the equation of the tangent to $y = \log_e(\tan x)$ at the point where $x = \dfrac{\pi}{4}$ **2**

Question 33

Arthur records the results of an experiment where the random variable X has the probability distribution shown below.

x	0	2	4	6	8	10
$P(X=x)$	0.12	0.17	0.25	0.21	0.15	0.1

a) Find the expected value **1**

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b) Find the variance **1**

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c) Calculate the standard deviation **1**

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Question 34

State the natural domain of the function $f(x)=\dfrac{1}{\sqrt{2-x^2}}$ **1**

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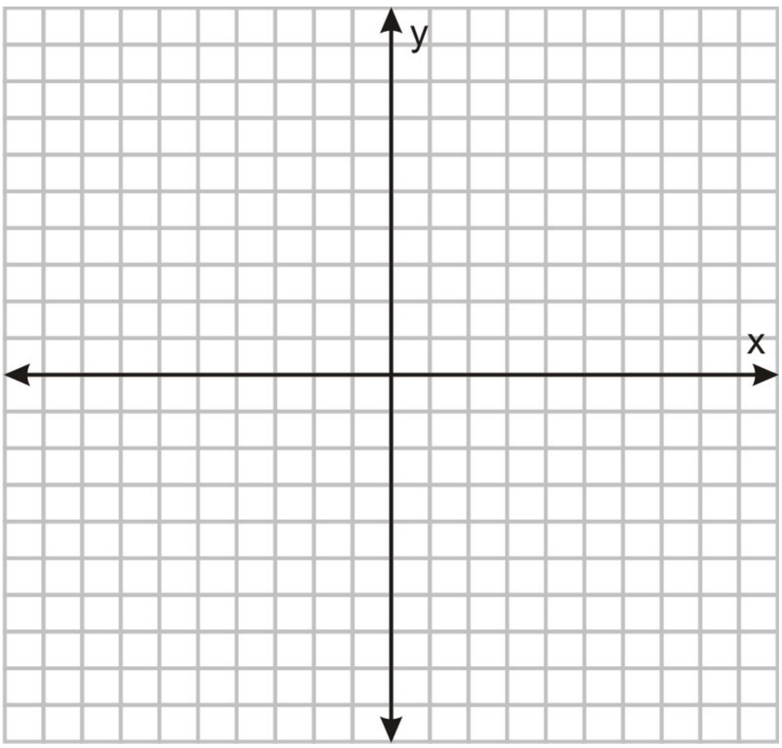
You are three-quarters through the exam (75 marks out of 100)

Question 35

Consider the function $y = 4x^3 - x^4$

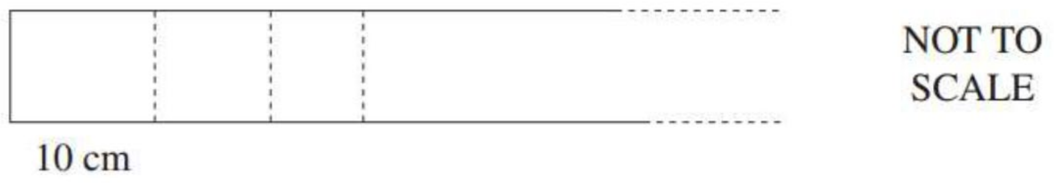
a) Find any stationary points and determine their nature and find any points of inflection **4**

b) Sketch the graph of the function, clearly showing the stationary points and intercepts with the coordinate axes. **2**



Question 36

Aurelia cuts rectangles of equal height from a strip of paper and arranges them in a row. The first rectangle has a length of 10 cm. The second rectangle has a length of 9.6 cm. The length of each subsequent rectangle is 96% of the length of the previous rectangle.



a) Find the length of the 5^{th} rectangular strip, correct to 3 significant figures **1**

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b) Find the length required to make the first 10 rectangles, correct to 3 significant figures **1**

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c) If Aurelia had a strip of paper 2.4 m in length, explain with working, if this is sufficient to make an unlimited number of rectangles. **1**

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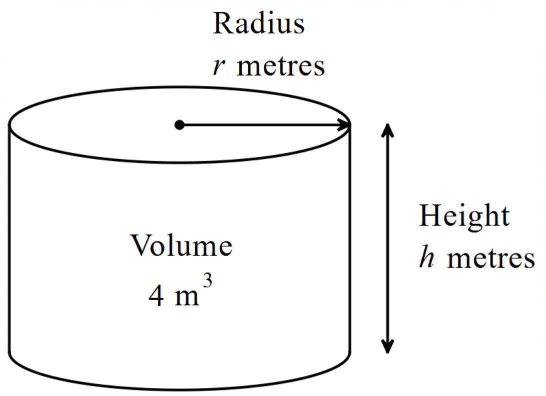
Question 37

A cylindrical water tank holds 4000 litres of water, making its volume 4 cubic metres as shown in the diagram (not to scale).



a) Show that the surface area is given by $S = 2\pi r^2 + 8r^{-1}$ **2**

b) Find the radius that would give the smallest possible surface area. **3**

Question 38

Simultaneously solve the equations $\log_{10}(2-x)-\log_{10} y = 1-\log_{10} 2$ and $\log_{10} 2x = \log_{10} y$ **2**

Question 39

Max borrows \$250 000 to purchase a house. He repays his loan in equal monthly instalments of \$$M$ for 25 years. Interest is charged at 3% p.a., compounded monthly.

a) By first finding expressions for A_1, A_2 and A_3 (the amount owing after 1 month, 2 months and 3 months), find an expression for A_n (the amount owing after n months). **2**

b) Find the amount of each monthly instalment, \$$M$, and hence calculate the amount of interest Max pays on the loan. **2**

Question 40

The acceleration of a particle is given by $\ddot{x} = 4\cos 2t$, where x is displacement in metres and t is time in seconds. Initially the particle is at the origin with a velocity of 1 ms^{-1}.

a) Show that the velocity of the particle is given by $\dot{x} = 2\sin 2t + 1$ **1**

b) Find the time when the particle first comes to rest. **2**

c) Find the displacement, x, of the particle in terms of t. **2**

End of Exam

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(A) 12.5 (B) $-151\frac{7}{8}$ (C) $101\frac{1}{4}$ (D) $-101\frac{1}{4}$

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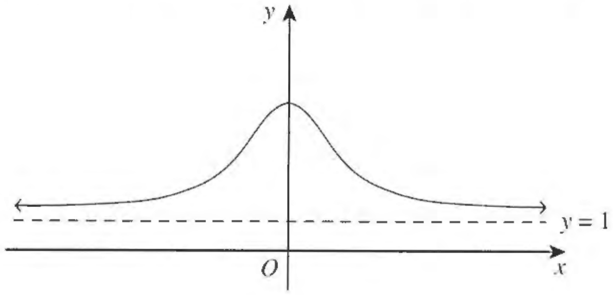

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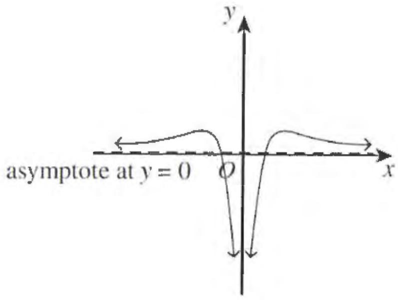

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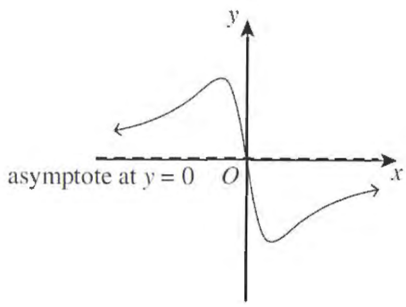

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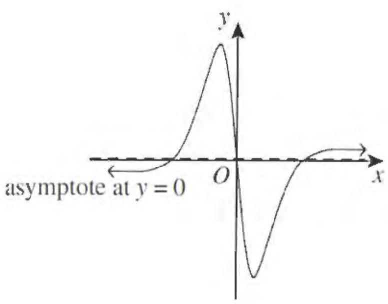

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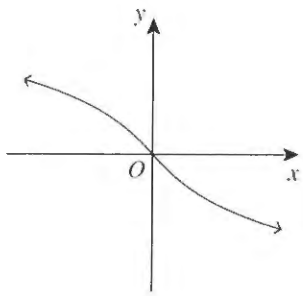

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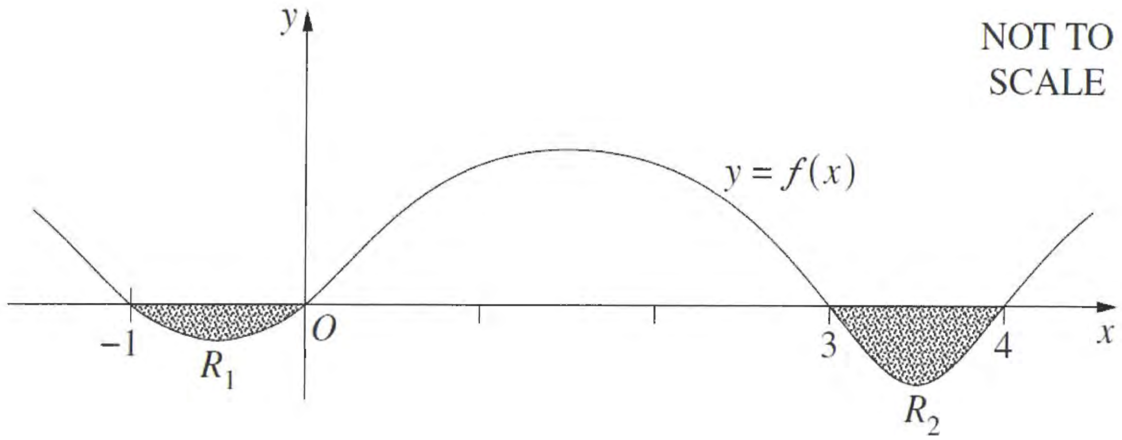

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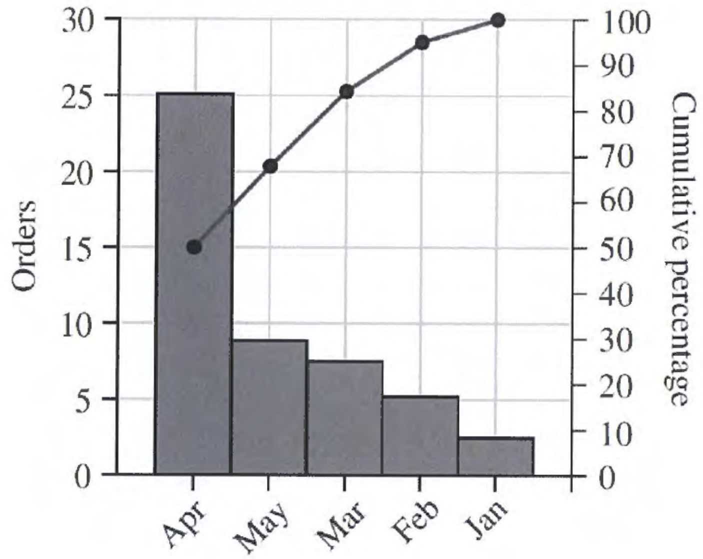

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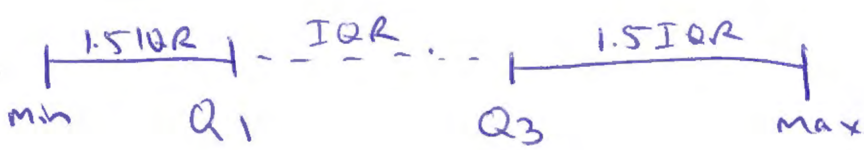
(D) The largest possible range is 5 times the interquartile range



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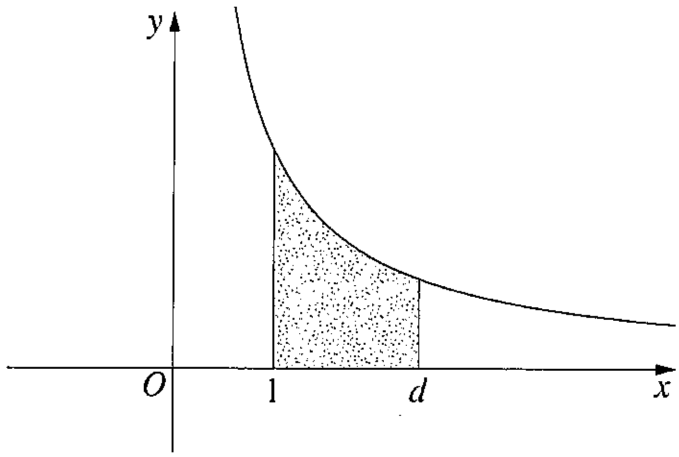

What value of d makes the shaded area equal to 2?

(A) e (B) $e + 1$ (C) $2e$ (D) e^2

End of Section I

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15, 18, 20, 20, 22, 26, 28, 32, 34, 47

a) Find the mean correct to one decimal place. **1**

26.2

b) Find the interquartile range. **1**

32 − 20 = 12

c) Is 47 an outlier for this set of data? Justify your answer with calculations. **1**

32 + 1.5 × 12 = 50

∴ 47 is not an outlier

Question 12

Evaluate $\sum_{r=1}^{6} r^r$ **1**

$1^1 + 2^2 + 3^3 + 4^4 + 5^5 + 6^6$

$= 50\,069$

Question 13

Annabelle takes 2 cans, without replacement, from a refrigerator that contains 8 cans of Pepsi, 4 cans of Coke and 3 cans of Sprite. Find the probability that Annabelle gets:

a) 2 of the same drink **1**

$\left(\frac{8}{15} \times \frac{7}{14}\right) + \left(\frac{4}{15} \times \frac{3}{14}\right) + \left(\frac{3}{15} \times \frac{2}{14}\right)$

$= \frac{37}{105}$

b) At least one Pepsi **1**

$1 - P(\text{no Pepsi}) = 1 - \left(\frac{7}{15} \times \frac{6}{14}\right)$

$= \frac{4}{5}$

Question 14

Solve $|7x - 12| \geq 16$ **2**

$7x - 12 \geq 16$ $\qquad$ $7x - 12 \leq -16$

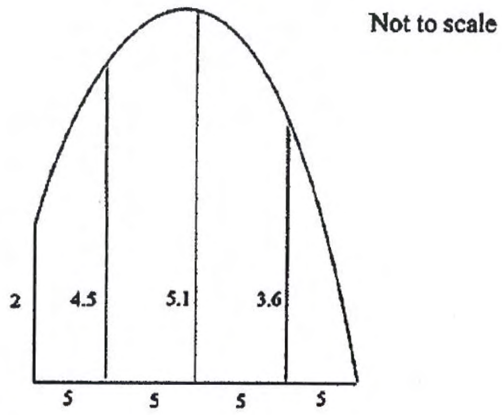
$7x \geq 28$ $\qquad$ $7x \leq -4$

$x \geq 4$ $\qquad$ $x \leq -\frac{4}{7}$

1 mark per correct solution

Question 15

Ronan is planting trees in the native garden which has dimensions as shown in the diagram. He can plant one tree per 3 square metres. Using the trapezoidal rule with 4 intervals, find the approximate area of the garden and therefore determine the number of trees he can plant. **2**



$\frac{5}{2}(2+0+2(4.5+5.1+3.6))$

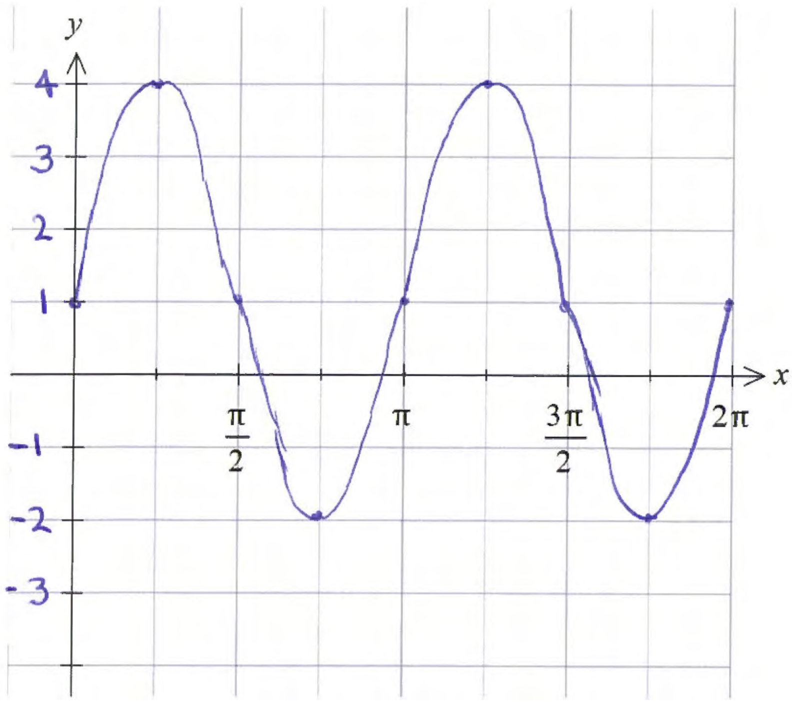
$= 71 \text{ m}^2$ (1 mark)

$\frac{71}{3} = 23\frac{2}{3}$

$\therefore$ Ronan can plant 23 trees (1 mark)

Question 16

Sketch the graph of $y = 3\sin 2x + 1$ for $0 \leq x \leq 2\pi$ **2**



−1 for each mistake
eg not shifting up 1 unit, not changing period correctly, incorrect amplitude.

Question 17

Jure owns a building supply company. They sell packets of screws which claim to have 75 screws per packet. To avoid customer complaints, the mean amount of screws in a packet is 79 and the variance is 4. (SD = 2)

a) What is the probability that a packet of screws selected randomly will have less than 75 screws in it? **1**

Z-score below −2

= 2.5%

packets packets screws

b) The business sets a target of less than 16% of all blocks will have more than 80. The business can change the mean weight of the boxes to meet this target. What is mean amount of screws required to meet the target if the standard deviation is 2? **1**

mean = 78

Question 18

a) $\int \cos x° dx$ **2**

$= \int \cos \frac{\pi x}{180} dx$ (1 mark change to radians)

$= \frac{1}{\pi/180} \sin \frac{\pi x}{180} + C$ (1 mark correct solution)

$= \frac{180}{\pi} \sin x° + C$

b) $\int_1^2 \frac{x}{2x^2+1} dx$ **2**

$= \frac{1}{4} \int_1^2 \frac{4x}{2x^2+1} dx$

$= \frac{1}{4}\left[\ln(2x^2+1)\right]_1^2$ (1 mark)

$= \frac{1}{4}\left[\ln(9) - \ln(3)\right]$

$= \frac{1}{4} \ln(3)$ (1 mark)

Question 19

Find $\frac{dy}{dx}$ and leave each answer as a single simplified fraction:

a) $y = (3x-4)\sqrt{5-2x}$ **2**

$y = (3x-4)(5-2x)^{1/2}$

$y' = (3x-4) \times \frac{1}{2}(5-2x)^{-\frac{1}{2}} \times -2 + (5-2x)^{1/2} \times 3$ (1 mark - correct use of product rule)

$= \frac{-3x+4}{(5-2x)^{1/2}} + 3(5-2x)^{1/2}$

$= \frac{-3x+4+3(5-2x)}{(5-2x)^{1/2}}$

$= \frac{-3x+4+15-6x}{\sqrt{5-2x}}$

$= \frac{19-9x}{\sqrt{5-2x}}$ (1 mark correct solution)

b) $y = \frac{\sin x}{1+\cos x}$ **2**

$y' = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$ (1 mark - correct use of quotient rule)

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{\cos x + 1}{(1+\cos x)^2}$

$= \frac{1}{1+\cos x}$ (1 mark - correct answer)

Question 20

The table below shows Luke's marks as well as the cohorts' mean and standard deviation across Science and English exams **2**

Subject	Luke's Mark	Mean	Standard Deviation
Science	86	73	8
English	83	74	5.5

In comparison to the cohort, in which of the subjects did Luke perform better? Justify your answer with relevant mathematical calculations.

Science Z-score $\frac{86-73}{8} = 1.625$ (1 mark for finding Z-scores)

English Z-score $\frac{83-74}{5.5} = 1.636$

$\therefore$ He performed better in English (1 mark)

Question 21

Find $\int_2^3 e^{5-2x}dx$, expressing your answer as a single simplified fraction **2**

$\left[-\frac{1}{2}e^{5-2x}\right]_2^3 = -\frac{1}{2}\left[e^{5-2x}\right]_2^3$

$= -\frac{1}{2}(e^{-1} - e)$ (1 mark)

$= \frac{-1}{2e} + \frac{e}{2}$

$= \frac{-1+e^2}{2e}$ (1 mark)

$= \frac{(e-1)(e+1)}{2e}$

Question 22

Jasmine's salary in her job increases by the same amount every year. In the 4^{th} year at her job she earned \$81 000. In the 9^{th} year of her job she earned \$98 500.

a) Find how much Jasmine earned in the first year of her job and by how much her pay increases each year. **2**

$a + 3d = 81\,000$
$a + 8d = 98\,500$
$5d = 17\,500$
$d = 3500$ (1 mark find a, 1 mark d)
$a + 10500 = 81000$
$a = 70\,500$
$\therefore$ 1st year pay = \$70 500
annual increase = \$3500

b) How much will Jasmine have earned in total after working at her job for 15 years? **1**

$S_{15} = \frac{15}{2}(2\times 70500 + 14\times 3500)$

$= \$1\,425\,000$

c) Bree started a job at the same time, and with the same starting salary as Jasmine. Bree's salary, however, increased by 2% per annum. With the use of logarithms, show in which year will her income first be greater than \$100 000? **2**

$70500(1.02)^{n-1} > 100\,000$
$1.02^{n-1} > 200/141$

$n - 1 > \frac{\log(\frac{200}{141})}{\log 1.02}$ (1 mark)

$n > 17.65 + 1$
$n > 18.65$
$\therefore$ She earns more than \$100 000 in her 19th year at the job. (1 mark)

Question 23

Prove that $\frac{\cos\theta}{1-\sin\theta} - \tan\theta = \sec\theta$ **3**

LHS $\frac{\cos\theta}{1-\sin\theta} - \frac{\sin\theta}{\cos\theta}$ (1 mark)

$= \frac{\cos^2\theta - \sin\theta(1-\sin\theta)}{(1-\sin\theta)\cos\theta}$

$= \frac{\cos^2\theta - \sin\theta + \sin^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{1-\sin\theta}{\cos\theta(1-\sin\theta)}$ (1 mark)

$= \frac{1}{\cos\theta}$

$= \sec\theta$ (1 mark)

$=$ RHS

Question 24

A school has two History classes that sit an exam. Andy's class has 16 students and has a mean mark of 65%, whilst Juliana's class has 24 students. The combined mean for the 2 classes is 68.6%. Find the mean for the Juliana's class. **2**

$\frac{16 \times 65 + 24x}{40} = 68.6$ (1 mark)

$1040 + 24x = 2744$

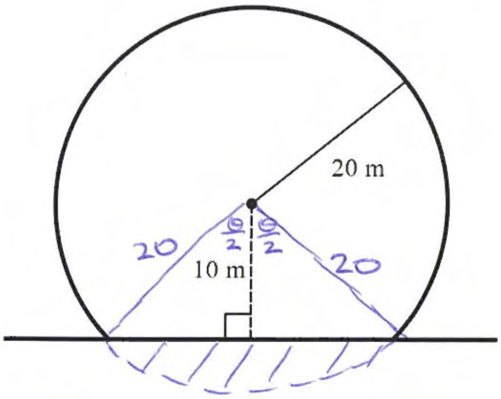
$24x = 1704$

$x = 71$

$\therefore$ mean of Juliana's class = 71% (1 mark)

Question 25

Peter ties his dog to a post with a 20 m long rope. On one side, the post is 10 m from a fence. Calculate the area correct to 4 significant figures, that the dog can play. **2**



Not to scale

Area = Circle − minor segment.

$= 400\pi - \frac{1}{2} \times 20^2 (\theta - \sin\theta)$ (1 mark)

$\cos\frac{\theta}{2} = \frac{10}{20}$ $\quad A = 400\pi - 200\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$\cos\theta/2 = 1/2$

$\frac{\theta}{2} = \frac{\pi}{3}$ $\quad = 400\pi - \frac{400\pi}{3} + 200\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \frac{2\pi}{3}$ $\quad = 1011\ m^2$ (1 mark)

Question 26

Use the table of values of $\phi(z)$ below to find $P(-0.9 \le Z \le 1.4)$ **2**

z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	first decimal place									
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

$P(Z \le 0.9) = P(Z \ge -0.9) = 0.8159$ (1 mark)

$P(Z \le 1.4) = 0.9192$

$\therefore P(Z \le -0.9) = 1 - 0.8159 = 0.1841$

$P(Z > 1.4) = 1 - 0.9192 = 0.0808$

$\therefore P(-0.9 \le Z \le 1.4) = 1 - 0.1841 - 0.0808 = 0.7351$

You are halfway through the exam (50 marks out of 100) (1 mark)

Question 27

A continuous random variable, T, represents the time taken in days to show symptoms after contracting a virus. It has the following probability density function.

$$f(t)=\begin{cases}\dfrac{k}{2t-1} & \text{for } 1\le t\le 14\\ 0 & \text{for } 0\le t<1 \text{ or } t>14\end{cases}$$

a) Show that $k=\dfrac{2}{\ln 27}$ **2**

$\int_1^{14} \frac{k}{2t-1} = 1$ (1 mark)

$\frac{k}{2}\int_1^{14} \frac{2}{2t-1} = 1$

$\frac{k}{2}\left[\ln(2t-1)\right]_1^{14} = 1$

$k(\ln 27 - \ln 1) = 2$

$k \ln 27 = 2$

$k = \frac{2}{\ln 27}$ (1 mark)

b) After how many days will a person have a 75% chance of showing symptoms after they have contracted the virus? **2**

$\int_1^{T} \frac{\frac{2}{\ln 27}}{2t-1}\,dt = \frac{3}{4}$

$\frac{2}{\ln 27}\times\frac{1}{2}\int_1^{T}\frac{2}{2t-1}\,dt = \frac{3}{4}$ (1 mark)

$\frac{1}{\ln 27}\left(\ln(2t-1)\right)_1^{T} = \frac{3}{4}$

$\ln(2T-1) - \ln 1 = \frac{3\ln 27}{4}$

$\ln(2T-1) = \frac{3\ln 27}{4}$

$2T-1 = e^{\frac{3\ln 27}{4}}$

$T = \frac{e^{\frac{3\ln 27}{4}}+1}{2}$

$T = 6.4$ days

$\therefore$ after 7 days (1 mark)

Question 28

Kyle recorded the number of push-ups and the number of sit-ups each of his classmates could do in a minute, as seen in the table below.

Push-Ups	Sit-Ups
8	18
10	17
17	22
22	30
29	25
36	47
40	50
48	48
51	57
60	81

a) The value of the correlation coefficient (r) is 0.95, explain what this means in the context of his data set. **1**

There is a strong positive correlation which means people who can do more push-ups can usually also do more sit-ups.

b) Use your calculator to find the equation of the least-squares regression line in the form of $y = Bx + A$. (Round each number to 1 decimal place.) **1**

$y = 1.1x + 4.3$

c) If Kyle is able to perform 32 sit-ups, use your equation in part b) to calculate the expected number of push-ups he could perform (to the nearest whole number). **1**

$32 = 1.1x + 4.3$

$27.7 = 1.1x$

$x = 25$

$\therefore$ Kyle is expected to do 25 push-ups

Question 29

Show that $\frac{d}{dx}\left[\ln\sqrt{\frac{2+x}{2-x}}\right]=\frac{2}{4-x^2}$ **3**

$\frac{d}{dx}\left[\ln(2+x)^{\frac{1}{2}}-\ln(2-x)^{\frac{1}{2}}\right]$ ← (1 mark)

$=\frac{d}{dx}\left[\frac{1}{2}\ln(2+x)^{\frac{1}{2}}-\frac{1}{2}\ln(2-x)^{\frac{1}{2}}\right]$

$=\frac{1}{2}\frac{d}{dx}\left[\ln(2+x)-\ln(2-x)\right]$

$=\frac{1}{2}\left[\frac{1}{2+x}+\frac{1}{2-x}\right]$

$=\frac{1}{2}\left(\frac{2-x+2+x}{(2+x)(2-x)}\right)$ (1 mark)

$=\frac{1}{2}\left(\frac{4}{4-x^2}\right)$

$=\frac{2}{4-x^2}$ (1 mark)

Question 30

A drug is used to control a medical condition. It is known that the quantity Q milligrams of drug remaining in the body after t hours satisfies an equation of the form $Q=Q_0e^{kt}$ where Q_0 and k are constants. The initial dose is 6 milligrams and after 15 hours the amount remaining in the body is half the initial dose.

a) Find the values of Q_0 and k. **2**

$Q_0=6$ (1 mark)

$Q=6e^{kt}$

$3=6e^{15k}$

$\frac{1}{2}=e^{15k}$

$15k=\log_e(\frac{1}{2})$ → $k=\frac{\log_e(\frac{1}{2})}{15}$ (1 mark)

b) After how long will one-eighth of the initial dose remain? **2**

$\frac{3}{4}=6e^{kt}$ (1 mark)

$\frac{1}{8}=e^{kt}$

$kt=\log_e(\frac{1}{8})$

$t=\frac{\log_e(\frac{1}{8})}{\frac{\log_e(\frac{1}{2})}{15}}$

$t=45$ hours (1 mark)

Question 31

Kloe has taken up golf, each year she plays 30 rounds of golf. In 2011 she totalled 3300 shots, in 2021 she totalled 2760 shots. The number of shots she took each year decreased by the same amount. To make it as a professional golfer she needs to get under 2115 shots in her 30 rounds. If this trend continues, in what year can she become a professional golfer? **3**

AP $a = 3300$ $a + 10d = 2760$

$10d = -540$

$d = -54$ (1 mark)

$a + (n-1)d < 2115$

$3300 + 54 - 54n < 2115$

$-54n < -1239$

$n > 22.944\ldots$

$\therefore n = 23$ years (1 mark)

$\therefore$ Kloe can be a professional golfer in 2033 (1 mark)

Question 32

a) Show that the derivative of $y = \log_e(\tan x)$ is $\frac{dy}{dx} = \tan x + \cot x$ **2**

$y' = \frac{\sec^2 x}{\tan x}$ (1 mark)

$= \frac{1 + \tan^2 x}{\tan x}$

$= \frac{1}{\tan x} + \frac{\tan^2 x}{\tan x}$

$= \cot x + \tan x$ (1 mark)

$= RHS$

b) Hence find the equation of the tangent to $y = \log_e(\tan x)$ at the point where $x = \frac{\pi}{4}$ **2**

at $x = \frac{\pi}{4}$

$m = \tan \pi/4 + \cot \pi/4$

$= 1 + 1$

$= 2$ (1 mark)

$y = \log_e(\tan \pi/4)$

$= \log_e(1)$

$= 0$

$y - 0 = 2(x - \pi/4)$

$y = 2x - \frac{\pi}{2}$ (1 mark)

Question 33

Arthur records the results of an experiment where the random variable X has the probability distribution shown below.

x	0	2	4	6	8	10	Sum
$P(X=x)$	0.12	0.17	0.25	0.21	0.15	0.1	1
$xP(x)$	0	0.34	1	1.26	1.2	1	4.8
$x^2P(x)$	0	0.68	4	7.56	9.6	10	31.84

a) Find the expected value **1**

4.8

b) Find the variance **1**

$31.84 - 4.8^2 = 8.8$

c) Calculate the standard deviation **1**

$\sqrt{8.8} = 2.966$ (3 d.p)

Question 34

State the natural domain of the function $f(x) = \dfrac{1}{\sqrt{2-x^2}}$ **1**

$-\sqrt{2} < x < \sqrt{2}$

or

$(-\sqrt{2}, \sqrt{2})$

You are three-quarters through the exam (75 marks out of 100)

Question 35

Consider the function $y = 4x^3 - x^4$

a) Find any stationary points and determine their nature and find any points of inflection **4**

$y' = 12x^2 - 4x^3$

$y'' = 24x - 12x^2$

$12x^2 - 4x^3 = 0$

$4x^2(3-x) = 0$

St.pt at $x=0$ and $x=3$

$(0,0)$ $(3,27)$ (1 mark)

at $x=0$ $y''=0$

$\therefore$ $(0,0)$ possible point of inflection.

at $(3,27)$ $y'' = -36$

$\therefore$ Maximum T.P.

(1 mark)

$y''=0$ to check POI

$12x(2-x)=0$

$x=0$, $x=2$

at $(2,16)$

x	1	2	2.5
y''	+12	0	-15

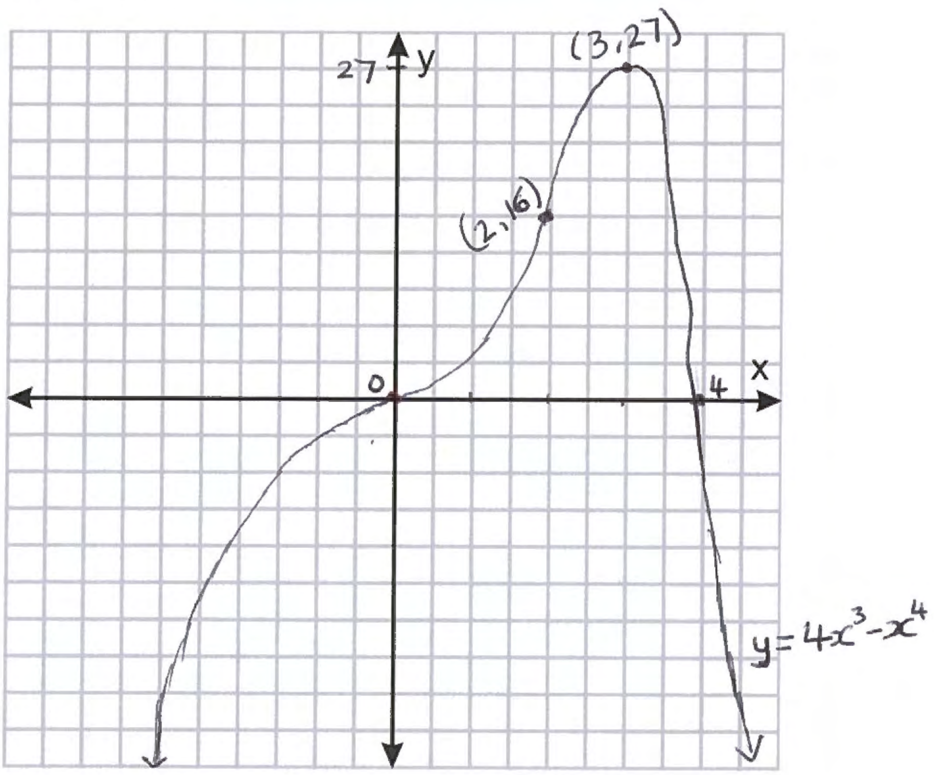
concavity changes $\therefore$ $(2,16)$ is point of inflection (1 mark)

x	-1	0	1
y''	-36	0	+12

concavity changes, $\therefore$ $(0,0)$ is a horizontal point of inflection.

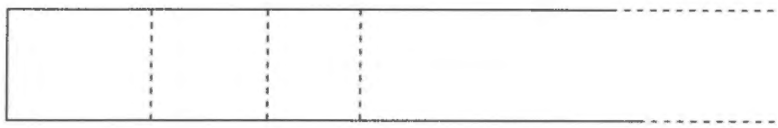
(1 mark)

b) Sketch the graph of the function, clearly showing the stationary points and intercepts with the coordinate axes. **2**



Question 36

Aurelia cuts rectangles of equal height from a strip of paper and arranges them in a row. The first rectangle has a length of 10 cm. The second rectangle has a length of 9.6 cm. The length of each subsequent rectangle is 96% of the length of the previous rectangle.



NOT TO SCALE

10 cm

a) Find the length of the 5^{th} rectangular strip, correct to 3 significant figures **1**

$a = 10 \quad r = 0.96$

$T_5 = 10 \times 0.96^4 = 8.49$ cm

b) Find the length required to make the first 10 rectangles, correct to 3 significant figures **1**

$S_{10} = \dfrac{10(1 - 0.96^{10})}{1 - 0.96}$

$= 83.8$ cm

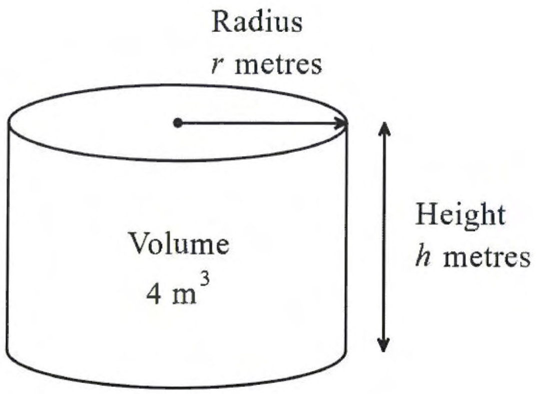
c) If Aurelia had a strip of paper 2.4 m in length, explain with working, if this is sufficient to make an unlimited number of rectangles. **1**

$S_\infty = \dfrac{10}{1 - 0.96} = 250$ cm

∴ 2.4m is NOT sufficient to make an unlimited number of rectangles.

Question 37

A cylindrical water tank holds 4000 litres of water, making its volume 4 cubic metres as shown in the diagram (not to scale).



a) Show that the surface area is given by $S = 2\pi r^2 + 8r^{-1}$ **2**

$V = \pi r^2 h$ $\quad S = 2\pi r^2 + 2\pi r h$

$4 = \pi r^2 h$ $\quad S = 2\pi r^2 + 2\pi r\left(\frac{4}{\pi r^2}\right)$

$h = \frac{4}{\pi r^2}$ $\quad S = 2\pi r^2 + \frac{8}{r}$

(1 mark)

$S = 2\pi r^2 + 8r^{-1}$ (1 mark)

b) Find the radius that would give the smallest possible surface area. **3**

$S' = 4\pi r - 8r^{-2}$

$4\pi r - \frac{8}{r^2} = 0$ (1 mark)

$\frac{4\pi r^3 - 8}{r^2} = 0$

$4\pi r^3 - 8 = 0$

$4\pi r^3 = 8$

$r = \sqrt[3]{\frac{8}{4\pi}}$

$r = \sqrt[3]{\frac{2}{\pi}}$ (1 mark)

$S'' = 4\pi + \frac{16}{r^3}$

at $r = \sqrt[3]{\frac{2}{\pi}}$ (or anywhere) $S'' > 0$

$\therefore$ Minimum area (1 mark)

Question 38

Simultaneously solve the equations $\log_{10}(2-x)-\log_{10} y = 1-\log_{10} 2$ and $\log_{10} 2x = \log_{10} y$ **2**

$\log_{10}(2-x)+\log_{10}2-1=\log_{10}y$

$\log_{10}(4-2x)-1=\log_{10}y$ --- ①

$\log_{10}2x=\log_{10}y$ --- ②

sub ② into ①

$\log_{10}(4-2x)-1=\log_{10}2x$

$\log_{10}\left(\frac{4-2x}{2x}\right)=1$

$\frac{4-2x}{2x}=10$

$\rightarrow 4-2x=20x$

$x=\frac{2}{11}$ (1 mark)

sub $x=2/11$ into ②

$y=\frac{4}{11}$ (1 mark)

Question 39

Max borrows \$250 000 to purchase a house. He repays his loan in equal monthly instalments of \$$M$ for 25 years. Interest is charged at 3% p.a., compounded monthly.

a) By first finding expressions for A_1, A_2 and A_3 (the amount owing after 1 month, 2 months and 3 months), find an expression for A_n (the amount owing after n months). **2**

$A_1 = 250000(1.0025)-M$

$A_2 = (250000(1.0025)-M)(1.0025)-m$

$= 250000(1.0025)^2 - m(1.0025)-m$

$= 250000(1.0025)^2 - m(1+1.0025)$ (1 mark)

$A_3 = 250000(1.0025)^3 - m(1+1.0025+1.0025^2)$

$\therefore A_n = 250000(1.0025)^n - m(1+1.0025+ \ldots +1.0025^{n-1})$

(1 mark)

b) Find the amount of each monthly instalment, \$$M$, and hence calculate the amount of interest Max pays on the loan. **2**

$A_{300}=0 \qquad 250000(1.0025)^{300} - m(1+1.0025 \ldots +1.0025^{299})=0$

$m = \dfrac{250000(1.0025)^{300}}{\left(\dfrac{1(1.0025^{300}-1)}{1.0025-1}\right)}$

$m = \$1185.53$ (1 mark)

$I = m \times 300 - 250000$

Interest $= \$105\,658.49$ (1 mark)

Question 40

The acceleration of a particle is given by $\ddot{x} = 4\cos 2t$, where x is displacement in metres and t is time in seconds. Initially the particle is at the origin with a velocity of 1 ms^{-1}.

a) Show that the velocity of the particle is given by $\dot{x} = 2\sin 2t + 1$ **1**

$\dot{x} = \int 4\cos 2t \, dt$

$\dot{x} = 2\sin 2t + C$

$\dot{x} = 1$ when $t = 0$

$1 = 2\sin 0 + C$

$C = 1$ $\therefore \dot{x} = 2\sin 2t + 1$

b) Find the time when the particle first comes to rest. **2**

$2\sin 2t + 1 = 0$

$2\sin 2t = -1$

$\sin 2t = -\frac{1}{2}$ (1 mark)

$\therefore 2t = \frac{7\pi}{6}$

$t = \frac{7\pi}{12}$ seconds. (1 mark)

c) Find the displacement, x, of the particle in terms of t. **2**

$d = \int 2\sin 2t + 1$

$= -\cos 2t + t + C$ (1 mark)

when $t = 0$ $d = 0$

$0 = -\cos 0 + 0 + C$

$0 = -1 + C$

$C = 1$

$\therefore d = -\cos 2t + t + 1$ (1 mark)

End of Exam

STUDENT NAME/NUMBER: ______________________________

Section I

10 Marks

Attempt Questions 1-10.

Allow about 15 minutes for this section.

Select either A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

This page must be handed in with your answer booklet.

	A	B	C	D
1			X	
2			X	
3		X		
4	X			
5			X	
6				X
7			X	
8				X
9			X	
10	X			